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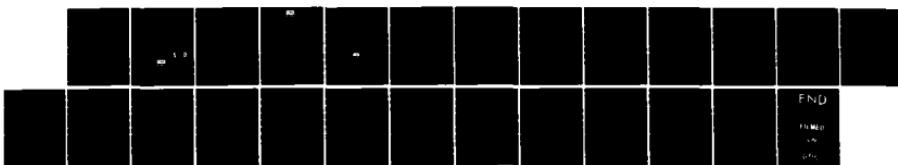
A MODEL FOR DRUG TESTING(U) CENTER FOR NAVAL ANALYSES
ALEXANDRIA VA RESOURCE ANALYSIS RESEARCH DEPT
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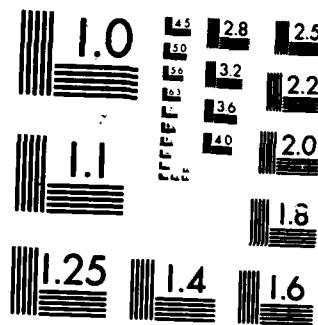
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RESEARCH MEMORANDUM

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A MODEL FOR DRUG TESTING

Peter Evanovich

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9 October 1985

MEMORANDUM FOR THE DEPUTY CHIEF OF STAFF FOR RESEARCH DEVELOPMENT
AND STUDIES

Subj: Center for Naval Analyses Research Memorandum 85-33

Encl: (1) CRM 85-33, "A Model for Drug Testing," by Peter Evanovich,
August 1985

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1. Enclosure (1) is forwarded as a matter of possible interest.
2. This Research Memorandum describes a drug testing model and explores ways in which the model can be used to assist decision makers in making policy with regard to a drug testing program. The model provided a conceptual framework for some of the analysis carried out in "The Effectiveness of Urinalysis as a Deterrent to Drug Use Study" which was conducted by MCOAG for the Deputy Chief of Staff for Manpower.

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A MODEL FOR DRUG TESTING

Peter Evanovich

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ABSTRACT

To make a drug-testing program successful and to minimize the cost of the program, the minimum number of tests that must be given in a specified period to identify a fixed percentage of drug users must be determined. This memorandum presents a Markov model that can be used to determine the number of tests that should be given. In addition, three applications of the model, showing how it can be used to analyze the drug-user population, are presented.

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INTRODUCTION

The ability of urine tests currently used to identify drug users depends on the level of the drug present in the subject's body when the test is administered. The level of drug in the body decreases over time after the substance is used. Thus, how often an individual uses a drug and the time lapse between drug use and a drug test are important variables in predicting how successful a testing program will be in identifying users. To minimize the cost of a program of drug testing, the minimum number of tests that must be given over a specified period in order to identify a set percentage of users in a population must be determined.

This memorandum introduces a Markov model developed to determine the minimum number of tests that should be given. Some of the assumptions made in developing the model impose certain limitations on its use. In particular, the model assumes that a drug test given by the Navy will have no "false positives"; that is, the test will not indicate drug use if given to a drug-free individual.

Applications of the model in analyzing the drug-user population are also presented. The model is a generic one, and the examples of its applications presented here are meant to illustrate its potential uses. Some of the assumptions made in the examples about the distribution of drug users in certain populations may not conform to reality. Most notable of these assumptions is that distribution of drug users among new recruits is identical to the distribution of drug users in the Navy at any given time. In any actual application, the model should be modified to reflect, to the extent possible, characteristics of the drug tests used and the populations being tested.

BASIC ASSUMPTIONS AND NOTATIONS

The Navy's current drug tests can detect a user who has taken the drug no more than a specified number of days before the test is administered. For purposes of this discussion, it is assumed that the test used will detect a user if he has used the drug in the last n days. Also, the probability of detection is assumed to be a function of the number of days since the drug was last used. In particular, $d(i)$, $i = 1, \dots, n$, will denote the probability that the drug will be detected by the test if the person being tested last used the drug i days ago.

The distribution of drug users within the group to be tested is also assumed to be known. In particular, it is assumed that users have been classified according to their usage rates—for example, once a day, once a week, or once a year. The number of distinct classes of users is assumed to be $m + 1$, denoted by $0, 1, 2, \dots, m$, and class 0 corresponds to the class of nonusers. The probability that an individual in class j , $j = 0, \dots, m$, will use the drug on an arbitrarily selected day will be denoted by $p(j)$. The probability that a person selected at random will be in class j will be denoted by $r(j)$, $j = 0, 1, \dots, m$. Note that the probability of a person being in class j , $j = 1, 2, \dots, m$, given that the person is a drug user, is

$$s(j) = \frac{r(j)}{r(1) + r(2) + \dots + r(m)} .$$

THE MARKOV MODEL

A finite-state Markov chain is used to estimate the probability of detection when the drug test is given at random to a person in one of the usage classes. The states of the Markov chain correspond to the number of days since the drug was last used. In particular, a person in class j , $j = 0, \dots, m$, can be in $n + 1$ states, $1, 2, \dots, n + 1$, where state i , $i = 1, \dots, n$, corresponds to use of the drug exactly i days ago, and state $n + 1$ corresponds to last use of the drug at least $n + 1$ days ago or never having used the drug.

If an individual from class j is in state i , $i = 1, \dots, n$, today, the probability that he will be in state 1 tomorrow is $p(j)$. The probability that he will be in state $i + 1$ tomorrow is $1 - p(j)$. The probability of his entering any state other than 1 or $i + 1$ is 0. If he is in state $n + 1$, the probability of entering state 1 tomorrow is $p(j)$, and the probability of being in state $n + 1$ tomorrow is $1 - p(j)$. The probability of going to a state other than 1 or $n + 1$ is 0. If $p(j)$ is not zero, the following Markov matrix results.

| Beginning state | Ending state | | | | | |
|-----------------|--------------|----------|----------|-------|----------|----------|
| | 1 | 2 | 3 | $n-1$ | n | $n+1$ |
| 1 | $p(j)$ | $1-p(j)$ | 0 | 0 | 0 | 0 |
| 2 | $p(j)$ | 0 | $1-p(j)$ | 0 | 0 | 0 |
| | | | | . | . | . |
| $n-1$ | $p(j)$ | 0 | 0 | 0 | $1-p(j)$ | 0 |
| n | $p(j)$ | 0 | 0 | 0 | 0 | $1-p(j)$ |
| $n+1$ | $p(j)$ | 0 | 0 | 0 | 0 | $1-p(j)$ |

This Markov matrix has the following steady-state probabilities associated with it:

$$x(1) = p(j)$$

$$x(2) = p(j)[1-p(j)]$$

$$x(3) = p(j)[1-p(j)]^2$$

$$x(4) = p(j)[1-p(j)]^3$$

.

.

.

$$x(n-1) = p(j)[1-p(j)]^{n-2}$$

$$x(n) = p(j)[1-p(j)]^{n-1}$$

$$x(n+1) = [1-p(j)]^n$$

If $j = 0$, then $p(j) = 0$, $x(i) = 0$ for $i = 1, \dots, n$, and $x(n+1) = 1$. Hence, when a test is administered to an individual from class $j = 1, \dots, m$, the

probability of detecting drugs in this individual is

$$D(j) = d(1)x(1) + \dots + d(n-1)x(n-1) + d(n)x(n)$$

$$D(0) = 0 .$$

The probability that an arbitrarily chosen user is detected is given by

$$Du = s(1)D(1) + s(2)D(2) + \dots + s(m-1)D(m-1) + s(m)D(m) .$$

Du can be interpreted as the detection rate among users. The probability that a person selected at random will show drug usage on a test (i.e., the rate of detection over the entire population) is given by

$$D = r(1)D(1) + r(2)D(2) + \dots + r(m-1)D(m-1) + r(m)D(m) .$$

MINIMAL-COST TESTING

The model developed above can be used to determine the smallest number of tests that can be given to each individual in the population to guarantee that a certain percentage of drug users will be identified. Let t , $0 < t < 1$, be the decimal equivalent of the percentage of actual drug users we wish to detect. Assuming that the tests will be administered to individuals randomly, the probability of detecting a user when k tests are administered to him is given by

$$1 - [1 - Du]^k .$$

This is the probability of at least one detection among k tests. To minimize the cost of testing to meet the requirement of identifying $100t$ percent of the users, the smallest k for which

$$1 - [1 - Du]^k \geq t$$

is selected. Then, k is the smallest nonnegative integer satisfying

$$k \geq \frac{\ln(1-t)}{\ln(1-Du)} .$$

Note that k can be viewed as the number of tests an individual is to be given over his lifetime in the service. In actuality, these tests would be given within a specified period (for example, a year), during which they could be administered in a random fashion to a relatively stable population.

Example 1

The following example illustrates the use of the model in establishing a testing program at minimal cost. For this illustration, arbitrary values are assigned to the variables.

The testing population is divided into five classes. The distribution of usage classes for the population is assumed to be known and to be as follows:

| Class | Usage rate | Percent of population |
|-------|-------------------|-----------------------|
| 0 | 0 (nonusers) | 75 |
| 1 | Once per month | 10 |
| 2 | Once per week | 8 |
| 3 | Twice per week | 5 |
| 4 | Once every 2 days | 2 |

This information can be used as follows to estimate the parameters of the model:

| Class (j) | Probability of use on arbitrary day $p(j)$ | Percent of population $r(j)$ | Percent of user population $s(j)$ |
|------------------|--|------------------------------------|---|
| 0 | 0 | 75 | 0 |
| 1 | $12/356 = 0.03$ | 10 | 40 |
| 2 | $1/7 = 0.14$ | 8 | 32 |
| 3 | $2/7 = 0.29$ | 5 | 20 |
| 4 | $1/2 = 0.50$ | 2 | 8 |

Note that we have interpreted the usage rate as the probability that a drug will be used on a randomly chosen day.

For this example, the test is assumed to be effective with a probability of 1 if the person being tested last used the drug 3 days at the most before the test is given to him.

Table 1 gives the steady-state probabilities, $x(i)$, and the detection rate for each class.

TABLE 1
STEADY-STATE PROBABILITIES FOR STATES
AND DETECTION RATES

| Class | States | | | | Probability of detection $D(j)$ |
|-------|-------------|-------------|-------------|-------------|---------------------------------------|
| | 1 $x(1)$ | 2 $x(2)$ | 3 $x(3)$ | 4 $x(4)$ | |
| 0 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 |
| 1 | 0.03 | 0.03 | 0.03 | 0.91 | 0.09 |
| 2 | 0.14 | 0.12 | 0.10 | 0.64 | 0.36 |
| 3 | 0.29 | 0.20 | 0.15 | 0.36 | 0.64 |
| 4 | 0.50 | 0.25 | 0.125 | 0.125 | 0.88 |

The detection rates for the entire population (D) and the user population (D_u) for a single test are

$$D = 0.09$$

$$D_u = 0.35 .$$

The desired number of tests per individual to identify 95 percent of users is the smallest nonnegative integer k for which

$$k \geq \frac{\ln(1 - 0.95)}{\ln(1 - 0.35)} = 6.95 .$$

Hence, we take $k = 7$.

Example 2

In the previous example, it was assumed that usage on any given day is described by a Bernoulli distribution whose mean for class j is the usage rate. It is interesting to compare the above results with those obtained when drug usage is assumed to follow another distribution. If it is assumed that usage occurs randomly (uniformly) over time, the time between consecutive uses is exponential. The mean of the exponential distribution describing the usage for class j , $j > 0$, will be the reciprocal of the usage rate for class j . Under these assumptions, the probabilities of use on an arbitrary day are the following:

| Class (j) | Probability of use on arbitrary day $p(j)$ | Percent of population $r(j)$ | Percent of user population $s(j)$ |
|------------------|--|------------------------------------|---|
| 0 | 0.00 | 75 | 0 |
| 1 | 0.03 | 10 | 40 |
| 2 | 0.13 | 8 | 32 |
| 3 | 0.25 | 5 | 20 |
| 4 | 0.39 | 2 | 8 |

The steady-state probabilities and the detection rates are given in table 2.

TABLE 2
STEADY-STATE PROBABILITIES FOR STATES AND DETECTION RATES

| Class | States | | | | Probability of detection $D(j)$ |
|-------|-------------|-------------|-------------|-------------|---------------------------------------|
| | 1 $x(1)$ | 2 $x(2)$ | 3 $x(3)$ | 4 $x(4)$ | |
| 0 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 |
| 1 | 0.03 | 0.03 | 0.03 | 0.91 | 0.09 |
| 2 | 0.14 | 0.12 | 0.10 | 0.64 | 0.36 |
| 3 | 0.25 | 0.19 | 0.14 | 0.42 | 0.58 |
| 4 | 0.39 | 0.24 | 0.15 | 0.22 | 0.78 |

The detection rates for the entire population (D) and the user population (D_u) are

$$D = 0.08$$

$$D_u = 0.34 .$$

The minimum number of tests per individual needed to detect 95 percent of the actual users is 8.

APPLICATIONS OF THE MODEL

Three additional applications of the Markov chain model are presented below. The applications are directed at determining the percentage of drug users in the population, estimating the particular usage class to which an individual belongs, and estimating the steady-state population in each usage class when a policy of discharging detected users is followed.

Estimating the Percentage of Drug Users in a Population

Drug testing is a deterrent to drug usage. Historical evidence suggests that when testing is used and detected drug users are penalized, the decrease in the percentage of the population in a particular class of users, other than the class of nonusers, is independent of the class. That is, if the population of class $j, j > 0$, decreases by x percent, then the population in every class other than 0 will also decrease by x percent.

Assume that the $r(j)$ values represent the decimal equivalent of the percentage of the population in each class before the testing program begins. After the program has begun, the percentages in each class are expected to change. Let $t(j), j = 0, 1, \dots, m$, denote the portion of the population in class j after program initiation. The assumption of proportionality implies that there is a constant c for which

$$t(j) = cr(j) \quad (j = 1, 2, \dots, m)$$

$$t(0) = 1 - [t(1) + t(2) + \dots + t(m)] .$$

If D is the detection rate when the distribution of users is given by $r(j)$ and D_0 is the detection rate when the distribution of users is given by $t(j)$, then

$$\begin{aligned}
 D_0 &= t(0)D(0) + t(1)D(1) + \dots + t(m)D(m) \\
 &= c[r(0)D(0) + r(1)D(1) + \dots + r(m)D(m)] \\
 &= cD ,
 \end{aligned}$$

and

$$c = \frac{D_0}{D} .$$

Example 3 (Example 1 continued)

Suppose that after some time has passed since testing began, the detection rate (D_0) is found to be 0.06 instead of the predicted (D) 0.08. Then,

$$c = \frac{0.06}{0.08} = 0.75 .$$

Using this value, it is possible to estimate the percentage of the population in each usage class. Table 3 gives the original percentages and the percentages for various detection rates arrived at over time after testing began.

TABLE 3
PERCENTAGE OF POPULATION IN CLASS j FOR VARIOUS DETECTION RATES

| Class (j) | p(j) | Detection rate | | | |
|------------------|----------|----------------|------|------|------|
| | | 0.08 | 0.06 | 0.04 | 0.02 |
| 0 | 0 | 75 | 81.2 | 88.0 | 93.4 |
| 1 | 1/2 | 10 | 7.5 | 5.0 | 2.4 |
| 2 | 1/7 | 8 | 6.0 | 4.0 | 2.0 |
| 3 | 2/7 | 5 | 3.8 | 2.5 | 1.3 |
| 4 | 12/365 | 2 | 1.5 | 1.0 | 0.5 |

Estimating the Usage Class of an Individual Based on Testing

Assume that k tests will be given to each individual in the population. How can the usage class to which an individual belongs be estimated based on the outcome of these tests? If an individual has been given $g \leq k$ tests and $0 \leq f \leq g$ of them have been positive, what can be concluded about the class to which this individual belongs?

The probability of an individual being in class j , given that f out of g tests given to him have been positive, $P(j)$, can be determined as follows:

$$P(j) = P(\text{in class } j \text{ given } f \text{ of } g \text{ tests positive})$$

$$= \frac{P(\text{in class } j \text{ and } f \text{ of } g \text{ tests positive})}{P(f \text{ of } g \text{ tests positive})}$$

$$= \frac{r(j) C(g, f) [D(j)^f] [1 - D(j)]^{g-f}}{u}$$

$$= \frac{r(j) [D(j)^f] [1 - D(j)]^{g-f}}{w},$$

where

$$u = r(0) C(g, f) D(0)^f [1 - D(0)]^{g-f} + \dots + r(n) C(g, f) D(n)^f [1 - D(n)]^{g-f}$$

and

$$w = r(0) \{ D(0)^f [1 - D(0)]^{g-f} + \dots + r(n) D(n)^f [1 - D(n)]^{g-f} \},$$

where $C(g, f)$ denotes the binomial coefficient " g binomial f ."

Example 4

Assume the parameters defined in example 1, which are repeated in the following table.

| Class (<i>j</i>) | Probability of use on arbitrary day <i>p(j)</i> | Percent of population <i>r(j)</i> | Percent of user population <i>s(j)</i> |
|-----------------------|---|---|--|
| 0 | 0 | 75 | 0 |
| 1 | $12/365 = 0.03$ | 10 | 40 |
| 2 | $1/7 = 0.14$ | 8 | 32 |
| 3 | $2/7 = 0.29$ | 5 | 20 |
| 4 | $1/2 = 0.50$ | 2 | 8 |

The detection rates are as follows:

| Class | States | | | | Probability of detection <i>D(j)</i> |
|-------|------------------|------------------|------------------|------------------|--|
| | 1 <i>x(1)</i> | 2 <i>x(2)</i> | 3 <i>x(3)</i> | 4 <i>x(4)</i> | |
| 0 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 |
| 1 | 0.03 | 0.03 | 0.03 | 0.91 | 0.09 |
| 2 | 0.14 | 0.12 | 0.10 | 0.64 | 0.36 |
| 3 | 0.29 | 0.20 | 0.15 | 0.36 | 0.64 |
| 4 | 0.50 | 0.25 | 0.125 | 0.125 | 0.88 |

Table 4 gives, for each class, the probabilities of an individual being in that class, based on 1 to 4 tests and the specific number of positive tests.

Policy Decisions

In using the model to estimate the percentage of drug users in a population, it was assumed that the punishment policy used to reprimand detected drug users resulted in a distribution of users among classes that maintained a fixed proportionality among the numbers in the various usage classes. The model can also be used to investigate how alternative policy decisions, where proportionality will not be maintained, will affect the distribution of the entire population across classes. In particular, the effect of discharging users who have had a specified number of positive tests within a given testing period is examined.

TABLE 4
PROBABILITY OF AN INDIVIDUAL BEING IN CLASS 0 TO 4, BASED ON TESTING

| Number of tests | Number of positive tests | Probability | | | | |
|-----------------|--------------------------|-------------|---------|---------|---------|---------|
| | | Class 0 | Class 1 | Class 2 | Class 3 | Class 4 |
| 1 | 0 | 0.822 | 0.100 | 0.056 | 0.020 | 0.003 |
| | 1 | 0.000 | 0.103 | 0.330 | 0.336 | 0.201 |
| 2 | 0 | 0.860 | 0.095 | 0.038 | 0.007 | 0.000 |
| | 1 | 0.000 | 0.203 | 0.458 | 0.286 | 0.052 |
| | 2 | 0.000 | 0.017 | 0.220 | 0.434 | 0.329 |
| 3 | 0 | 0.884 | 0.089 | 0.025 | 0.003 | 0.000 |
| | 1 | 0.000 | 0.315 | 0.499 | 0.175 | 0.011 |
| | 2 | 0.000 | 0.044 | 0.400 | 0.444 | 0.112 |
| | 3 | 0.000 | 0.002 | 0.122 | 0.429 | 0.446 |
| 4 | 0 | 0.901 | 0.082 | 0.016 | 0.001 | 0.000 |
| | 1 | 0.000 | 0.428 | 0.476 | 0.094 | 0.002 |
| | 2 | 0.000 | 0.086 | 0.545 | 0.341 | 0.029 |
| | 3 | 0.000 | 0.008 | 0.271 | 0.536 | 0.186 |
| | 4 | 0.000 | 0.000 | 0.062 | 0.386 | 0.552 |

Assume that the distribution of drug users across the classes of people entering the Navy from the civilian population is known. Let $t(j), j = 0, \dots, m$ denote the probability that a new inductee will be in user class j .

Let w represent the probability that an arbitrarily selected individual will leave the service during the period being used for reasons other than the drug detection policy described below.

Also assume that in any given period, k tests will be administered and that any person detected h out of k , $h \leq k$, times in this period will be discharged from the Navy.

The probability of detection within each class can be used to compute the probability of having at least h out of k tests positive for a randomly selected individual in that class. This probability is denoted by $P(j)$, $j = 0, \dots, m$.

If $r(j)$, $j = 0, \dots, m$, denotes the percentages of the population in classes $0, \dots, m$, respectively at steady state (i.e., after the policy has been put into effect and the classes have stabilized), then the rate at which individuals will leave class j is given by

$$P(j)r(j) + w[1 - P(j)]r(j) = \{P(j) + w[1 - P(j)]\}r(j) = B(j)r(j)$$

where

$$B(j) = P(j) + w[1 - P(j)] .$$

The rate at which individuals will leave the service is given by

$$X = B(0)r(0) + B(1)r(1) + \dots + B(m)r(m) .$$

The rate at which people leave the service is assumed to be equal to the rate at which they enter, so that the service population is constant. Since it is assumed that $r(j)$ represents the steady-state percentages in each usage class, the rate of entry into each usage class must be equal to the exit rate for each usage class. That is,

$$t(j)X = B(j)r(j) \quad (j = 0, \dots, m) .$$

This set of m equations for the $m + 1$ unknowns $r(j)$, $j = 0, \dots, m$ has one redundant equation. However, $r(j)$ must also satisfy the requirement that

$$r(0) + r(1) + \dots + r(m) = 1 ,$$

which leads to the following system of equations:

$$[\alpha(0)-1]B(0)r(0) + \alpha(0)B(1)r(1) + \dots + \alpha(0)B(m-1)r(m-1) + \alpha(1)B(m)r(m) = 0$$

$$\alpha(1)B(0)r(0) + [\alpha(1)-1]B(1)r(1) + \dots + \alpha(1)B(m-1)r(m-1) + \alpha(1)B(m)r(m) = 0$$

.

.

$$\alpha(m-1)B(0)r(0) + \alpha(m-1)B(1)r(1) + \dots + [\alpha(m-1)-1]B(m-1)r(m-1)$$

$$+ \alpha(m-1)B(m)r(m) = 0$$

$$r(0) + r(1) + r(m-1) + r(m) = 1 .$$

Let

$$A(j) = B(0) \dots B(j-1)B(j+1) \dots B(m) \quad (j = 1, \dots, m) .$$

Then the solution of this system is given by

$$r(j) = \frac{\alpha(j)A(j)}{[\alpha(0)A(0) + \dots + \alpha(m)A(m)]} \quad (j = 0, \dots, m) .$$

Example 5 (Example 1 continued)

The rate at which people leave the population for reasons not related to drug detection is assumed to be $w = 0.1$ (i.e., 10 percent of the population will leave if no new drug policy is followed). It is also assumed that new inductees

obey the distribution of drug users described in example 1, which is as follows:

| Class (j) | Probability of use on arbitrary day $p(j)$ | Percent of inductee population $t(j)$ | Percent of inductee user population |
|--------------|--|---|--|
| 0 | 0 | 75 | 0 |
| 1 | $12/365 = 0.03$ | 10 | 40 |
| 2 | $1/7 = 0.14$ | 8 | 32 |
| 3 | $2/7 = 0.29$ | 5 | 20 |
| 4 | $1/2 = 0.50$ | 2 | 8 |

The following table gives the detection rates for the usage classes for a single test:

| Class | States | | | | Probability of detection $D(j)$ |
|-------|-------------|-------------|-------------|-------------|---------------------------------------|
| | 1 $x(1)$ | 2 $x(2)$ | 3 $x(3)$ | 4 $x(4)$ | |
| 0 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 |
| 1 | 0.03 | 0.03 | 0.03 | 0.91 | 0.09 |
| 2 | 0.14 | 0.12 | 0.10 | 0.64 | 0.36 |
| 3 | 0.29 | 0.20 | 0.15 | 0.36 | 0.64 |
| 4 | 0.50 | 0.25 | 0.125 | 0.125 | 0.88 |

Now suppose that seven tests are given at random over a period of one year, and the policy is to discharge anyone who is detected on at least two of the tests. The probabilities of discharge for each class, $P(j)$, are

$$P(0) = 0.000$$

$$P(1) = 0.125$$

$$P(2) = 0.783$$

$$P(3) = 0.826$$

$$P(4) = 1.000 .$$

Note that

$$B(0) = 0 + (1 - 0)(0.1) = 0.1000$$

$$B(1) = 0.125 + (1 - 0.125)(0.1) = 0.2125$$

$$B(2) = 0.8047$$

$$B(3) = 0.8434$$

$$B(4) = 0.1000$$

$$A(0) = (0.2125)(0.8047)(0.8434)(1) = 0.1442$$

$$A(1) = (0.1000)(0.8047)(0.8434)(1) = 0.0679$$

$$A(2) = 0.0179$$

$$A(3) = 0.0171$$

$$A(4) = 0.0144$$

$$\alpha(0)A(0) = (0.75)(0.1442) = 0.1082$$

$$\alpha(1)A(1) = (0.10)(0.0679) = 0.0068$$

$$\alpha(2)A(2) = 0.0014$$

$$\alpha(3)A(3) = 0.0009$$

$$\alpha(4)A(4) = 0.0003$$

and

$$\alpha(0)A(0) + \dots + \alpha(4)A(4) = 0.1175 .$$

Hence, the steady-state percentages in the usage classes are

$$r(0) = 0.1082/0.1175 = 0.921$$

$$r(1) = 0.0068/0.1175 = 0.058$$

$$r(2) = 0.012$$

$$r(3) = 0.007$$

$$r(4) = 0.003 .$$

Summary

These applications of the Markov model illustrate the potential of the model for analyzing a population in terms of drug usage. Given a drug-testing program, the model can be used to estimate the distribution of drug users in the population on the basis of observed detection rates and to identify the usage class of individuals based on testing results. Finally, the last example shows how the model can be used to predict the effects of policy decisions on drug use.

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